Properties of Triangles

Question1

In a $\triangle ABC$, if $r_1 = 4, r_2 = 8$ and $r_3 = 24$, then $a : b : c =$

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Options:

A.

$4 : 7 : 9$

B.

$2 : 3 : 5$

C.

$1 : 2 : 6$

D.

$6 : 2 : 1$

Answer: A

Solution:

We have, $r_1 = 4, r_2 = 8$ and $r_3 = 24$

$$\therefore \quad 4 = \frac{\Delta}{s-a}, 8 = \frac{\Delta}{s-b} \text{ and } 24 = \frac{\Delta}{s-c}$$

$$\Rightarrow \quad s-a = \frac{\Delta}{4} \quad \ldots(i)$$

$$s-b = \frac{\Delta}{8} \quad \ldots(ii)$$

$$\text{And} \quad s-c = \frac{\Delta}{24} \quad \ldots(iii)$$

On adding these three equations, we get

$$3s-(a+b+c)=\frac{\Delta}{4}+\frac{\Delta}{8}+\frac{\Delta}{24}$$
$$\Rightarrow 3s-2s=\frac{6\Delta+3\Delta+\Delta}{24}=\frac{10\Delta}{24}\Rightarrow s=\frac{5}{12}\Delta$$

From Eq. (i), $a=s-\frac{\Delta}{4}=\frac{5\Delta}{12}-\frac{\Delta}{4}=\frac{2\Delta}{12}=\frac{\Delta}{6}$

From Eq. (ii), $b=s-\frac{\Delta}{8}=\frac{5}{12}\Delta-\frac{\Delta}{8}=\frac{7}{24}\Delta$

From Eq. (iii), $c=s-\frac{\Delta}{24}=\frac{5}{12}\Delta-\frac{\Delta}{24}=\frac{9}{24}\Delta$

$\therefore a:b:c=\frac{\Delta}{6}:\frac{7}{24}\Delta:\frac{9}{24}\Delta=4:7:9$

Question2

In a $\triangle ABC,\left(r_2+r_3\right)\operatorname{cosec}^2\left(\frac{A}{2}\right)=$

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Options:

A.

$4R\cot\left(\frac{A}{2}\right)$

B.

$2R\cot^2\left(\frac{A}{2}\right)$

C.

$\frac{4R}{\tan^2\left(\frac{A}{2}\right)}$

D.

$\frac{2R}{\tan\left(\frac{A}{2}\right)}$

Answer: C

Solution:

We know that

$$\begin{aligned} r_2 &= 4R\cos\tfrac{A}{2}\sin\tfrac{B}{2}\cos\tfrac{C}{2} \\ r_3 &= 4R\cos\tfrac{A}{2}\cos\tfrac{B}{2}\sin\tfrac{C}{2} \end{aligned}$$

$\therefore \quad (r_2+r_3) = 4R\cos\frac{A}{2}\left[\sin\frac{B}{2}\cos\frac{C}{2}+\cos\frac{B}{2}\sin\frac{C}{2}\right]$

$\Rightarrow \quad (r_2+r_3) = 4R\cos\frac{A}{2}\sin\left(\frac{B+C}{2}\right)$

$\Rightarrow \quad (r_2+r_3) = 4R\cos\frac{A}{2}\sin\left(90^\circ-\frac{A}{2}\right) = 4R\cos\frac{A}{2}\cos\frac{A}{2}$

$$\begin{aligned} \therefore (r_2+r_3)\operatorname{cosec}^2\left(\frac{A}{2}\right) &= 4R\cos^2\frac{A}{2}\operatorname{cosec}^2\frac{A}{2} \\ &= \frac{4R}{\tan^2\frac{A}{2}} \end{aligned}$$

Question3

If p_1, p_2, p_3 are the altitudes and $a=4, b=5, c=6$ are the sides of a $\triangle ABC$, then $\frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=$

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Options:

A.

$\frac{77}{225}$

B.

$\frac{44}{225}$

C.

$\frac{308}{225}$

D.

$\frac{22}{75}$

Answer: B

Solution:

Given, $a=4, b=5$ and $c=6$

Since, p_1, p_2 and p_3 are altitudes of the sides a, b and c respectively.

$\therefore \quad p=\frac{2\Delta}{\text{base}}$

$\Rightarrow \quad p_1=\frac{2\Delta}{a}, p_2=\frac{2\Delta}{b}, p_3=\frac{2\Delta}{c}$

$\Rightarrow \quad \frac{1}{p_1^2}=\frac{a^2}{4\Delta^2}, \frac{1}{p_2^2}=\frac{b^2}{4\Delta^2}, \frac{1}{p_3^2}=\frac{c^2}{4\Delta^2}$

$\therefore \quad \frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}$

$=\frac{a^2}{4\Delta^2}+\frac{b^2}{4\Delta^2}+\frac{c^2}{4\Delta^2}$

$=\frac{a^2+b^2+c^2}{4\Delta^2}$

We know that $s=\frac{a+b+c}{2}$

$=\frac{4+5+6}{2}=\frac{15}{2}$

So, $\quad \Delta=\sqrt{s(s-a)(s-b)(s-c)}$

(Using heron's formula)

$$=\sqrt{\frac{15}{2} \cdot \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2}}$$
$$=\sqrt{\frac{1575}{16}}=\frac{\sqrt{1575}}{4}$$
$$=\Delta^2=\frac{1575}{16}$$

And $a^2+b^2+c^2=4^2+5^2+6^2$

$=16+25+36=77$

$$So, \frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=\frac{77}{4\Delta^2}$$
$$=\frac{77}{4\times\frac{1575}{16}}=\frac{77}{\frac{6300}{16}}$$
$$=\frac{77\times 16}{6300}=\frac{44}{225}$$

Question4

Let the angles A, B, C of a $\triangle ABC$ be in arithmetic progression. If the exradii r_1, r_2, r_3 of $\triangle ABC$ satisfy the condition $r_3^2=r_1 r_2+r_2 r_3+r_3 r_1$, then $b=$

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Options:

A.

$\frac{2a}{\sqrt{3}}$

B.

$\sqrt{2}a$

C.

$\sqrt{3}a$

D.

a

Answer: C

Solution:

Given angles A, B, C are in AP and $A+B+C=180^\circ \quad (\because$ angles of $\triangle ABC)$

So, $\frac{A+C}{2}=B$

$\therefore 2B+B=180^\circ$

$$\Rightarrow \quad 3B=180^\circ$$
$$\Rightarrow \quad B=60^\circ$$

Given that $r_3^2=r_1r_2+r_2r_3+r_3r_1$

Since, $B=60^\circ$, using the cosine rule for side b.

$$b^2=a^2+c^2-2ac\cos 60^\circ$$
$$=a^2+c^2-2ac\cdot\frac{1}{2}=a^2+c^2-ac$$

We know that, $s^2=r_1r_2+r_2r_3+r_3r_1$

And given, $r_3^2=r_1r_2+r_2r_3+r_3r_1$

So, $r_3^2=s^2$

$\Rightarrow r_3=s$

Since, $r_3=\frac{\Delta}{s-c}$

$$\Rightarrow \quad s=\frac{\Delta}{s-c}\Rightarrow\Delta=s(s-c)$$
$$\Rightarrow \quad \sqrt{s(s-a)(s-b)(s-c)}=s(s-c)$$

Squaring both sides, we get

$$\Rightarrow \quad s(s-a)(s-b)(s-c)=s^2(s-c)^2$$
$$\Rightarrow \quad (s-a)(s-b)=s(s-c)$$
$$\Rightarrow \quad s^2-sa-sb+ab=s^2-sc$$
$$\Rightarrow \quad ab=s(a+b-c)$$
$$\Rightarrow \quad ab=\frac{(a+b+c)}{2}\cdot(a+b-c)$$
$$\left[\because s=\frac{a+b+c}{2}\right]$$
$$\Rightarrow \quad 2ab=(a+b)^2-c^2$$
$$\Rightarrow \quad 2ab=a^2+b^2+2ab-c^2$$
$$\Rightarrow \quad a^2+b^2-c^2=0$$
$$\Rightarrow \quad c^2=a^2+b^2$$

So, the triangle is a right-angled triangle with right angle at c.

But, we have $B=60^\circ$ and $C=90^\circ$

So, $A=180^\circ-60^\circ-90^\circ=30^\circ$

Using the sine rule,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$\Rightarrow \quad \frac{a}{\sin 30^\circ}=\frac{b}{\sin 60^\circ}$$
$$\Rightarrow \quad \frac{a}{\frac{1}{2}}=\frac{b}{\frac{\sqrt{3}}{2}}\Rightarrow 2a=\frac{2b}{\sqrt{3}}$$
$$\Rightarrow \quad \sqrt{3}a=b$$
$$\therefore \quad b=\sqrt{3}a$$

Question5

In a $\triangle ABC$, if $a=5, b=3, c=7$, then $\sqrt{\frac{\sin(A-B)}{\sin(A+B)}}=$

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Options:

A. $\frac{4}{7}$

B. 16

C. 36

D. $\frac{4}{5}$

Answer: A

Solution:

$a=5, b=3, c=7$

$\cos A=\frac{9+49-25}{2 \cdot 3 \cdot 7}=\frac{11}{14}$

$\cos B=\frac{25+49-9}{2 \cdot 5 \cdot 7}=\frac{65}{2 \cdot 5 \cdot 7}=\frac{13}{14}$

$\frac{\sin (A-B)}{\sin (A+B)}=\frac{\sin A \cos B-\cos A \cdot \sin B}{\sin (\pi-C)}$

$=\frac{\sin A \cos B-\cos A \sin B}{\sin C}$

$=\frac{a \cos B-b \cos A}{c}=\frac{5 \times \frac{13}{14}-3 \times \frac{11}{14}}{7}$

$=\frac{65-33}{14 \times 7}=\frac{32}{14 \times 7}=\frac{16}{49}$

$\Rightarrow \sqrt{\frac{\sin (A-B)}{\sin (A+B)}}=\sqrt{\frac{16}{49}}=\frac{4}{7}$

Question6

In a $\triangle ABC$, if $r_1=6, r_2=9, r_3=18$, then $\cos A=$

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Options:

A. $\frac{5}{13}$

B. $\frac{4}{5}$

C. $\frac{5}{7}$

D. $\frac{7}{25}$

Answer: B

Solution:

$r_1 = 6, r_2 = 9, r_3 = 18,$

$\frac{r_1}{r_2} = \frac{s-b}{s-a} = \frac{6}{9} = \frac{2}{3}$

$\frac{r_2}{r_3} = \frac{s-c}{s-b} = \frac{1}{2}, \frac{r_3}{r_1} = \frac{s-a}{s-c} = 3$

$\Rightarrow \quad \frac{a+c-b}{b+c-a} = \frac{2}{3}, \frac{s-c}{s-b} = \frac{1}{2}, \frac{s-a}{s-c} = 3$

$\Rightarrow \quad \frac{a+c-b}{b+c-a} = \frac{2}{3}, \frac{a+b+c}{a+c-b} = \frac{1}{3} \frac{b+c-a}{a+b-c} = 3$

$\Rightarrow \quad 4a + 2b = 4c$

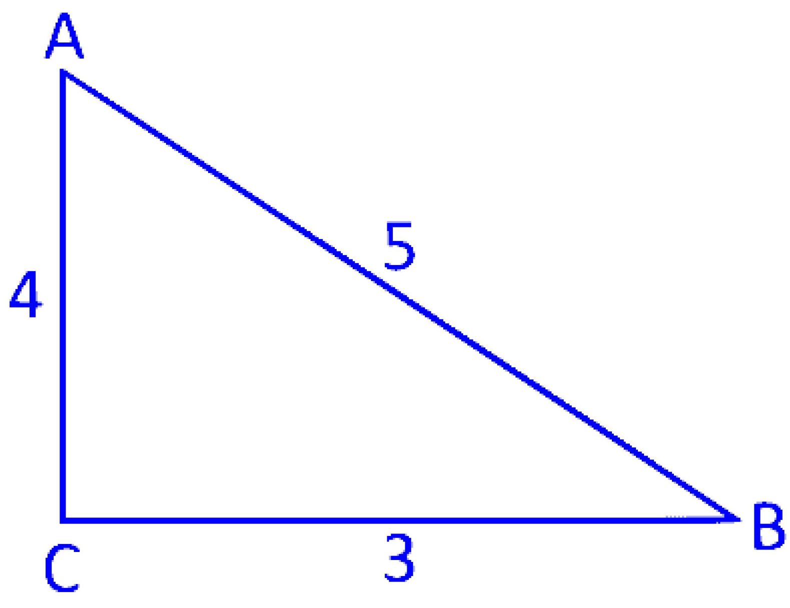
$\Rightarrow \quad 2a + b = 2c \quad \ldots \text{(i)}$

$\Rightarrow \quad 5a + c = 5b \quad \ldots \text{(ii)}$

$a + 3b = 3c \quad \ldots \text{(iii)}$

On solving, we get

$a : b : c = 3 : 4 : 5$



$\angle C = \frac{\pi}{2} \Rightarrow \cos A = \frac{4}{5}$

Question7

If ABC is an isosceles triangle with base BC, then $r_1 =$

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Options:

A. $R^2 \cos^2 A$

B. $\frac{a^2}{2}$

C. $\frac{r}{R}$

D. $R^2 \sin^2 A$

Answer: D

Solution:

Given that $\triangle ABC$ is an isosceles triangle with base BC, we want to find r_1.

The known formulas are:

$r = \frac{\Delta}{S}$

$r_1 = \frac{\Delta}{s-a}$

Where r is the inradius, r_1 is the exradius opposite the side a, Δ is the area of the triangle, and S is the semi-perimeter.

Let's explore the product $r \cdot r_1$:

$r \cdot r_1 = \frac{\Delta^2}{s(s-a)} = \frac{s(s-a)(s-b)(s-c)}{s(s-a)}$

Since the triangle is isosceles with $b = c$:

$r \cdot r_1 = (s-b)(s-c) = (s-b)^2$

In the context of an isosceles triangle:

$s = \frac{a+b+c}{2}$

Since $b = c$, we have:

$s - b = \frac{a+b+c}{2} - b = \frac{a+c-b}{2}$

Thus:

$(s-b)^2 = \left(\frac{a+c-b}{2}\right)^2 = \frac{a^2}{4}$

Rewriting this in terms of the circumcircle radius R:

$\frac{a}{\sin A} = 2R$

Therefore:

$r \cdot r_1 = \frac{a^2}{4} = \frac{4R^2 \sin^2 A}{4} = R^2 \sin^2 A$

Hence, for the isosceles triangle $\triangle ABC$ with base BC, we find:

$r_1 = R^2 \sin^2 A$

Question8

In $\triangle ABC$, if $r_1 + r_2 = 3R, r_2 + r_3 = 2R$, then

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Options:

A. ABC is a right-angled isosceles triangle

B. $B = \frac{\pi}{3}$

C. $A = 90^{\circ}, a \neq b \neq c$

D. $C = 90^{\circ}, a : b : c = 2 : 1 : \sqrt{3}$

Answer: C

Solution:

Given data indicates:

$r_1 + r_2 = 3R$

$r_2 + r_3 = 2R$

Let's analyze these conditions:

From $r_1 + r_2 = 3R$:

We start by substituting in the formula:

$\frac{\Delta}{s-a} + \frac{\Delta}{s-b} = \frac{3abc}{4\Delta}$

Simplifying, we have:

$\Delta^2 \left(\frac{1}{s-a} + \frac{1}{s-b}\right) = \frac{3abc}{4}$

Using the identity:

$s(s-b)(s-c) + s(s-a)(s-c) = \frac{3abc}{4}$

Combining terms:

$s(s-c)(s-b+s-a) = \frac{3abc}{4}$

Further simplification:

$s(s-c)c = \frac{3abc}{4}$

Resulting relationship:

$s(s-c) = \frac{3ab}{4}$

Therefore:

$\sqrt{\frac{s(s-c)}{ab}} = \frac{\sqrt{3}}{2}$

Which implies:

$\cos\left(\frac{C}{2}\right) = \frac{\sqrt{3}}{2}$

Hence,

$\frac{C}{2} = 30^\circ \quad \text{or} \quad C = 60^\circ$

From $r_2 + r_3 = 2R$:

Use:

$s(s-a) = \frac{2bc}{4}$

Gives:

$\sqrt{\frac{s(s-a)}{bc}} = \frac{1}{\sqrt{2}}$

Therefore:

$\cos\left(\frac{A}{2}\right) = \frac{1}{\sqrt{2}}$

Thus,

$\frac{A}{2} = 45^\circ \quad \text{or} \quad A = 90^\circ$

Final Angles of the Triangle:

$\angle A = 90^\circ$

$\angle B = 60^\circ$

$\angle C = 30^\circ$

This implies the triangle does not fit typical ratios like $a : b : c = 2 : 1 : \sqrt{3}$; instead, the lengths of sides must be inconsistent, i.e., $a \neq b \neq c$, which corresponds to a non-equilateral, non-isosceles right triangle.

Question9

In a $\triangle ABC$, the sides b, c are fixed. In measuring angle A, if there is an error of δA, then the percentage error in measuring the length of the side a is

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Options:

A. $\frac{2\Delta\delta A}{R\sin A} \times 100$

B. $2 \times \frac{\delta A}{A} \times 100$

C. $\frac{\Delta\delta A}{2R^2\sin^2 A} \times 100$

D. $\frac{\Delta^2\delta A}{R\sin A} \times 100$

Answer: C

Solution:

In a triangle $\triangle ABC$, consider the scenario where the sides b and c are fixed. When measuring angle A, if there is a small error denoted by δA, we wish to find the percentage error in the length of side a.

The cosine rule for angle A is given by:

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

Rearranging, we have:

$2bc\cos A = b^2 + c^2 - a^2$

Taking the derivative with respect to angle A introduces an error term:

$-2bc\sin A \cdot \delta A = -2a \cdot \delta a$

Simplifying gives:

$bc\sin A \cdot \delta A = a \cdot \delta a$

Thus, the infinitesimal change in a is:

$\delta a = \frac{bc\sin A \cdot \delta A}{a}$

Using the sine rule:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

This implies:

$a = 2R\sin A$

To find the percentage error in a, we calculate:

$\frac{\delta a}{a} \times 100 = \frac{\frac{bc\sin A \cdot \delta A}{a}}{a} \times 100$

Substituting for a:

$= \frac{\frac{1}{2}bc\sin A \cdot \delta A}{\frac{1}{2}(2R\sin A)^2} \times 100$

This simplifies to:

$= \frac{\Delta \cdot \delta A}{2R^2\sin^2 A} \times 100$

where $\Delta = \frac{1}{2}bc\sin A$.

Question10

In triangle ABC, if $a = 4, b = 3$ and $c = 2$, then $2(a - b\cos C)(a - c\sec B) =$

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Options:

A. 0

B. 1

C. 2

D. 3

Answer: D

Solution:

Given, $a = 4, b = 3, c = 2$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

$= \frac{16+9-4}{2\times4\times3} = \frac{21}{24} = \frac{7}{8}$

$\cos B = \frac{a^2+c^2-b^2}{2\times a\times c} = \frac{16+4-9}{2\times4\times2} = \frac{11}{16}$

$2(a - b\cos C)(a - (c\sec B))$

$2\left(4 - \frac{3\times4}{8}\right)\left(4 - \frac{2\times16}{11}\right)$

$= 2\left(4 - \frac{21}{8}\right)\left(4 - \frac{32}{11}\right)$

$= 2\left(\frac{32-21}{8}\right)\left(\frac{44-32}{11}\right) = \frac{11}{4} \times \frac{12}{11} = 3$

Question11

In $\triangle ABC$, if $A=45^{\circ}, C=75^{\circ}$ and $R=\sqrt{2}$, than $r=$

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Options:

A. $\frac{3+\sqrt{3}}{\sqrt{3}+\sqrt{2}+1}$

B. $\frac{\sqrt{3}+1}{\sqrt{3}+\sqrt{2}+1}$

C. $\frac{\sqrt{3}+1}{\sqrt{6}+\sqrt{3}+3}$

D. $\frac{\sqrt{3}+1}{\sqrt{3}+\sqrt{2}}$

Answer: B

Solution:

Given, If $A=45^{\circ}, C=75^{\circ}$,

$R=\sqrt{2}$, then $r=?$

We know that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$

$\frac{a}{\sin 45^{\circ}}=\frac{b}{\sin 60^{\circ}}=\frac{c}{\sin 75^{\circ}}=2\sqrt{2}$

$a\sqrt{2}=\frac{2b}{\sqrt{3}}=\frac{2\sqrt{2}c}{\sqrt{3}+1}=2\sqrt{2}$

$a=2, b=\sqrt{6}, c=\sqrt{3}+1$

$a+b+c=2s \Rightarrow s=\frac{3+\sqrt{3}+\sqrt{6}}{2}$

$\therefore r=(s-b)\tan\frac{B}{2}$

$=\left[\frac{3+\sqrt{3}+\sqrt{6}}{2}-\sqrt{6}\right]\tan 30^{\circ}$

$=\frac{3+\frac{\sqrt{3}-\sqrt{6}}{2\sqrt{3}}=\frac{\sqrt{3}[\sqrt{3}+1-\sqrt{2}]}{2\sqrt{3}}}{2}$

$=\frac{(\sqrt{3}+1-\sqrt{2})(\sqrt{3}+1+\sqrt{2})}{2[\sqrt{3}+1+\sqrt{2}]}$

$= \frac{3+1+2\sqrt{3}-2}{2(\sqrt{3}+1+\sqrt{2})} = \frac{(\sqrt{3}+1)}{(\sqrt{3}+\sqrt{2}+1)}$

Question12

If $A(1,2,-3), B(2,3,-1)$ and $C(3,1,1)$ are the vertices of $\triangle ABC$, then $\left|\frac{-\cos A}{\cos B}\right| =$

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Options:

A. $\frac{3\sqrt{3}}{4\sqrt{2}}$

B. $\frac{3\sqrt{3}}{\sqrt{7}}$

C. $\frac{4\sqrt{2}}{3\sqrt{3}}$

D. $\frac{\sqrt{7}}{3\sqrt{3}}$

Answer: B

Solution:

Given, $A \equiv (1,2,-3), B \equiv (2,3,-1)$

$C \equiv (3,1,1)$

$AB = \sqrt{1+1+4} = \sqrt{6} = c$

$AC = \sqrt{4+1+16} = \sqrt{21} = b$

$BC = \sqrt{1+4+4} = \sqrt{9} = a$

$\left|\frac{\cos A}{\cos B}\right| = \frac{\frac{b^2+c^2-a^2}{2bc}}{\frac{a^2+c^2-b^2}{2ac}} = \left|\frac{a\left(b^2+c^2-a^2\right)}{b\left(a^2+c^2-b^2\right)}\right|$

$= \left|\frac{\sqrt{9}}{\sqrt{21}} \frac{(21+6-9)}{(9+6-21)}\right| = \left|\frac{\sqrt{3}}{\sqrt{7}} \times \frac{18}{(-6)}\right|$

Therefore, $\left|\frac{\cos A}{\cos B}\right| = \frac{3\sqrt{3}}{\sqrt{7}}$

Question13

If $A+B+C=2S$, then
$\sin(S-A)\cos(S-B)-\sin(S-C)\cos S=$

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Options:

A. $\cos A\sin B\sin C$

B. $\sin A\cos B\cos C$

C. $\cos A\sin B$

D. $\sin A\cos B$

Answer: C

Solution:

Given, $A+B+C=2S$

Now, $\sin(S-A)\cos(S-B)+\sin(S-C)\cos S$

$=\frac{1}{2}[2\sin(S-A)\cos(S-B)+2\sin(S-C)\cos S]$

$=\frac{1}{2}[\sin(2S-A-B)+\sin(B-A)$

$+\sin(2S-C)+\sin(-C)]$.

$=\frac{1}{2}[\sin C-\sin(A-B)+\sin(A+B)-\sin C]$

$=\frac{1}{2}[\sin(A+B)-\sin(A-B)]$

$=\frac{1}{2}(2\cos A\sin B)=\cos A\sin B$

Question14

In a $\triangle ABC$, if $\tan\frac{A}{2}:\tan\frac{B}{2}:\tan\frac{C}{2}=15:10:6$, then $\frac{a}{b-c}=$

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Options:

A. $\frac{8}{3}$

B. $\frac{7}{3}$

C. 5

D. 4

Answer: D

Solution:

We have, in a triangle ABC,

$\tan\frac{A}{2} : \tan\frac{B}{2} : \tan\frac{C}{2} = 15 : 10 : 6$

We know that

$$r = \frac{\Delta}{s} = (s-a)\tan\frac{A}{2}$$

$$= (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$$

$$\therefore \frac{\tan\frac{A}{2}}{\tan\frac{B}{2}} = \frac{s-b}{s-a} = \frac{15}{10} = \frac{15}{10} \times \frac{2}{2} = \frac{30}{20}$$

$$\text{and } \frac{\tan\frac{B}{2}}{\tan\frac{C}{2}} = \frac{s-c}{s-b} = \frac{10}{6} = \frac{10}{6} \times \frac{5}{5} = \frac{50}{30}$$

$$\therefore \quad (s-a) : (s-b) : (s-c)$$

$$= 20 : 30 : 50 = 2 : 3 : 5$$

$$\Rightarrow a : b : c = 8 : 7 : 5$$

Let $a = 8k, b = 7k$ and $c = 5k$

$$\text{Now, } \frac{a}{b-c} = \frac{8k}{7k-5k} = \frac{8k}{2k} = 4$$

Question15

In a $\triangle ABC$, $\frac{a(r_1+r_2r_3)}{r_1-r+r_2+r_3}=$

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Options:

A. $\sqrt{\pi_1 r_2 r_3}$

B. $r_1r_2+r_2r_3+r_3r_1$

C. $2(R+r)$

D. $2+\frac{r}{2R}$

Answer: A

Solution:

$$\frac{a\left(r_1+r_2r_3\right)}{r_1-r+r_2+r_3}$$

$$=\frac{a\left(\frac{\Delta}{s}\cdot\frac{\Delta}{s-a}+\frac{\Delta}{s-b}\cdot\frac{\Delta}{s-c}\right)}{\left(r_1+r_2+r_3\right)-r}$$

$$=\frac{a\left(\Delta^2\cdot\left(\frac{(s-b)(s-c)+s(s-a)}{s(s-a)(s-b)(s-c)}\right)\right)}{(4R-r)+r}$$

$$=\frac{a\left(\Delta^2\cdot\frac{s^2-(a+b+c)s+bc+s^2}{\Delta^2}\right)}{4R}$$

$$=\frac{a}{4R}\left(s^2-(2s)s+bc+s^2\right)$$

$$=\frac{abc}{4R}=\Delta=\sqrt{r_1r_2r_3}$$

Question16

In a $\triangle ABC$, if $(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}=a^2+b^2$, then $\cos A=$

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Options:

A. $\cos B$

B. $\sin B$

C. $\sin C$

D. $\cos C$

Answer: C

Solution:

We have,

$$(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2} = a^2 + b^2$$

$$\left(a^2+b^2-2ab\right) \cos^2 \frac{C}{2} + \left(a^2+b^2+2ab\right)$$

$$\sin^2 \frac{C}{2} = a^2 + b^2$$

$$a^2+b^2+2ab\left(\sin^2 \frac{C}{2} - \cos^2 \frac{C}{2}\right) = a^2 + b^2$$

$$2ab\left(\sin^2 \frac{C}{2} - \cos^2 \frac{C}{2}\right) = 0$$

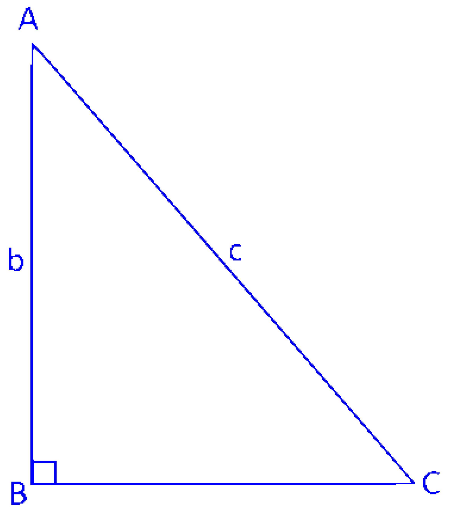
$$\Rightarrow \quad \sin^2 \frac{C}{2} - \cos^2 \frac{C}{2} = 0$$

$$\Rightarrow \quad \tan^2 \frac{C}{2} = 1$$

$$\Rightarrow \quad \tan \frac{C}{2} = \pm 1$$

$$\Rightarrow \quad \frac{C}{2} = 45$$

$$\Rightarrow \quad C = 90^{\circ}$$



So, $\cos A = \frac{b}{c} \Rightarrow \sin B = \frac{b}{c}$

$\Rightarrow \quad \cos A = \sin B$

Question17

In a $\triangle ABC$, if $r_1 r_2 + r_3 = 35, r_2 r_3 + r_1 = 63$ and $r_3 r_1 + r_2 = 45$, then $2s =$

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Options:

A. 28

B. 21

C. 25

D. 36

Answer: C

Solution:

$\because r_1 r_2 + \pi_3 = 35$

$$\Rightarrow \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} + \frac{\Delta}{s} \cdot \frac{\Delta}{s-c} = 35$$

$$\Rightarrow \quad \Delta^2\left(\frac{s \cdot (s-c)+(s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right)=35$$

$$\Rightarrow \quad 2s^2-(a+b+c)s+ab=35$$

$$[\because s(s-a)(s-b)(s-c)=\Delta^2]$$

$$\Rightarrow 2s^2-2s^2+ab=35[\because a+b+c=25]$$

$$\Rightarrow \quad ab=35 \quad \ldots \text{(i)}$$

$$\because \quad r_2r_3+\pi_1=63$$

So, $bc=63 \quad \ldots$ (ii)

$$\because \quad r_3r_1+r \cdot r_2=45$$

So, $ca=45 \quad \ldots$ (iii)

From Eq. (i), (ii) and (iii), we get

$a=5, b-7, c=9$

Hence, $2s=a+b+c=5+7+9=21$

Question18

PQR is an isosceles triangle with $PQ=PR$. If the radius of the circumcircle of $\triangle PQR$ is equal to the length of PQ then $\angle P=$

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Options:

A. 30°

B. 60°

C. 45°

D. 120°

Answer: D

Solution:

Given, PQR is an isosceles triangle, with $PQ = QR$. and also the radius of circumcircle of $\triangle PQR$ is equal to the length of PQ,

So, $PQ = PR = r$ (say) and $\angle Q = \angle R$

So, sign rule $\frac{PQ}{\sin \angle R} = 2r$

$$\Rightarrow \frac{n}{\sin \angle R} = 2r \Rightarrow \sin \angle R = \frac{1}{2} = \sin \frac{\pi}{6}$$
$$\therefore \angle R = \frac{\pi}{6}$$

Now, $\angle P = \pi - (\angle Q + \angle R) = \pi - \left(\frac{\pi}{6} + \frac{\pi}{6}\right)$

$$= \pi - \frac{\pi}{3}$$
$$= \frac{2\pi}{3} = \frac{2 \times 180^\circ}{3} = 120^\circ$$

Question19

In $\triangle ABC$, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos \cdot C}{c}$ and side $a = 2$, then area of the $\triangle ABC$ (in sq units) is

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Options:

A. $8\sqrt{2}$

B. $4\sqrt{3}$

C. $\sqrt{3}/2$

D. $\sqrt{3}$

Answer: D

Solution:

Given, $\triangle ABC$ and

$\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$ (given)

We know that, $a=2R\sin A, b=2R\sin B$, and $c=2R\sin C$

$$\Rightarrow \frac{\cos A}{2R\sin A}=\frac{\cos B}{2R\sin B}=\frac{\cos C}{2R\sin C}$$
$$=\tan A=\tan B=\tan C$$

So, this know that the $\triangle ABC$ is an equilateral triangle.

Thus, $a=b=c=2$ (obviously)

$$\therefore \text{ Area } =\frac{\sqrt{3}}{4}(a)^2=\frac{\sqrt{3}}{4}\times(2)^2$$
$$=\sqrt{3} \text{ sq units}$$

Question20

In an isosceles right angled triangle, a straight line is drawn from the mid-point of one of the equal sides to the opposite vertex. Then, a pair of possible values of the cotangents of the two angles so formed at that vertex are

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Options:

A. 1 and 2

B. 2 and 3

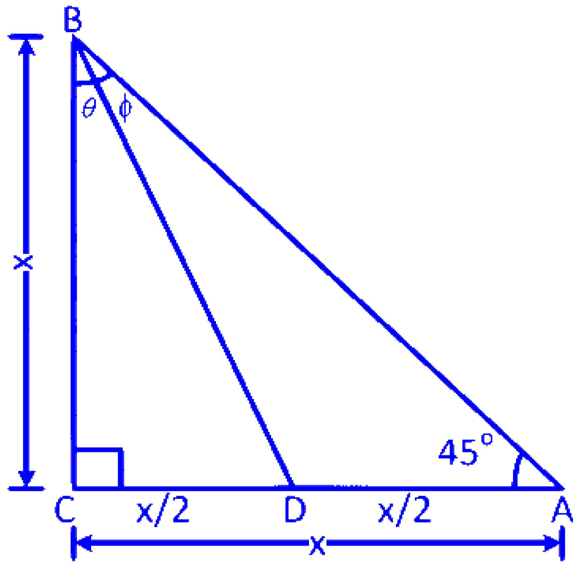
C. 3 and 4

D. 4 and 5

Answer: B

Solution:

$\theta+\phi=45^{\circ}$...(i)



In $\triangle BCD$,

$$\tan\theta = \frac{CD}{BC} = \frac{\frac{x}{2}}{x} = \frac{1}{2}$$
$$\Rightarrow \quad \cot\theta = 2$$
$$\phi = 45^\circ - \theta \qquad \text{[from Eq. (i)]}$$
$$\Rightarrow \quad \tan\phi = \tan(45^\circ - \theta)$$
$$= \frac{\tan 45^\circ - \tan\theta}{1 + \tan 45^\circ \cdot \tan\theta}$$
$$= \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{3}$$
$$\Rightarrow \quad \cot\phi = 3$$

$\therefore$ Possible values of the cotangents of the two angles are 2 and 3.

Question21

In a $\triangle ABC$, if $r_1 = 2r_2 = 3r_3$, then $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} =$

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Options:

A. $\frac{75}{60}$

B. $\frac{155}{60}$

C. $\frac{176}{60}$.

D. $\frac{191}{60}$

Answer: D

Solution:

Given, $r_1 = 2r_2 = 3r_3$

Let $r_1 = 2r_2 = 3r_3 = k$

$r_1 = k$

$$\Rightarrow \quad \frac{\Delta}{s-a} = k \Rightarrow s - a = \frac{\Delta}{k}$$

$$a = s - \frac{\Delta}{k}$$

$2r_2 = k$

$$\Rightarrow \frac{2\Delta}{s-b} = k \Rightarrow s - b = \frac{2\Delta}{k}$$

$$\Rightarrow \quad b = s - \frac{2\Delta}{k}$$

$$\Rightarrow \quad 3r_3 = k$$

$$\frac{3\Delta}{s-c} = k \Rightarrow s - c = \frac{3\Delta}{k}$$

$$\Rightarrow \quad c = s - \frac{3\Delta}{k}$$

Adding Eqs. (i), (ii) and (iii), we get

$$a + b + c = 3s - \frac{6\Delta}{k}$$

$$\Rightarrow \quad 2s = 3s - \frac{6\Delta}{k} \Rightarrow s = \frac{6\Delta}{k}$$

$$a = \frac{5\Delta}{k}, b = \frac{4\Delta}{k} \text{ and } c = \frac{3\Delta}{k}$$

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = \frac{5}{4} + \frac{4}{3} + \frac{3}{5}$$

$$= \frac{75 + 80 + 36}{60} = \frac{191}{60}$$
